Math 2414 **Graded Homework 5 (Section 11.6, 11.8)** **Fall 2019**

Instructions:

1) **Due during the first 10 minutes of your problem session the week of Dec. 2, 2019**
2) **Assignments must be submitted in a blue book – no exceptions. Bluebooks are available at the on campus/off campus bookstore, student union, etc.**
3) **On the front cover, write the following at the top:**
   a) **TA name**
   b) **Your name**
   c) **Assignment name**
   d) **Problem session number**
4) **You may use both sides of each page, but start a new problem at the top of a new side.**
5) **Solutions must be presented neatly, completely, and with logical flow.**
6) **15% will be deducted for assignments turned in after the first 10 minutes of class.**
7) **25% will be deducted for assignments which are not neat and orderly.**
8) **15% will be deducted for assignments without your TA's name.**
9) **Assignments will not be accepted after class.**

1) Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

a. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4+1}$

b. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n}{n^4+1}$

c. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n n^2}{n^3+1}$

d. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+4}{3^n}$

e. $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1} 1\cdot 4\cdot 7\cdots(3k-2)}{k!\,2^k}$

f. $\sum_{n=1}^{\infty} \frac{(2n^2+1)^n}{(-5)^{n+1} n^{2n}}$

g. $\sum_{n=1}^{\infty} \left(\frac{(2n+1)^2}{4n^2} \right)^{n^2}$

2) Find the radius of convergence and the interval of convergence of each power series. If the radius is 0, then state the value of x at which it converges.

a. $\sum_{n=1}^{\infty} \frac{5^n x^n}{n^3}$

b. $\sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^n}{6^n n}$

c. $\sum_{k=1}^{\infty} \frac{x^k}{2 \cdot 4 \cdot 6 \cdot \cdots \cdot (2k)}$

d. $\sum_{n=1}^{\infty} \frac{(-1)^n (x+4)^{n+1}}{5^n \sqrt{n+1}}$

e. $\sum_{n=1}^{\infty} \frac{n!(x+3)^n}{n^2+1}$

3) Later in this course, we will learn that the function, $\arctan x$, is equivalent to a power series for x on the interval $-1 \le x \le 1$:

$$\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

We can use this power series to approximate the constant π.

a) First, evaluate $\arctan(1)$. (You do not need the series to evaluate it.)
b) Use your answer from part (a) and the power series above to find a series representation for π. (The answer will be just a series – not a power series.)
c) Verify that the series you found in part (b) converges.
d) Use your convergent series from part (b) to approximate π with |error| < 0.5.
e) How many terms would you need to approximate π with |error| < 0.001?