**Math 2414** **Graded Homework 3 (Section 10.1 - 10.4)** **Fall 2019**

**Instructions:**

1) **Due: During the first 10 minutes of your problem session the week of Oct. 21, 2019**
2) **Assignments must be submitted in a blue book – no exceptions. Bluebooks are available at the on campus/off campus bookstore, student union, etc.**
3) **On the front cover, write the following at the top:**
   a) **TA name**
   b) **Your name**
   c) **Assignment name**
   d) **Problem session number**
4) **You may use both sides of each page, but start a new problem at the top of a new side.**
5) **Solutions must be presented neatly, completely, and with logical flow.**
6) **15% will be deducted for assignments turned in after the first 10 minutes of class.**
7) **25% will be deducted for assignments which are not neat and orderly.**
8) **15% will be deducted for assignments without your TA's name.**
9) **Assignments will not be accepted after class.**

1. Consider the parametric equations

$$x = \cos\left(\frac{t}{2}\right) + \sin\left(\frac{t}{2}\right),\ \ y = \frac{1}{\sqrt{2}}\cos\left(\frac{t}{2}\right) - \frac{1}{\sqrt{2}}\sin\left(\frac{t}{2}\right), \qquad \frac{\pi}{2} \le t \le \frac{9\pi}{2}$$

   a. Eliminate the parameter $t$ to find a Cartesian equation for the parametric curve. Hint: Multiply $y$ by $\sqrt{2}$, then square both equations and add them together.
   b. Sketch the parametric curve, indicating with arrows the direction in which the curve is traced.

2. Consider the parametric curve

$$x = (t^2 - 2t + 1)e^{2t},\ \ y = (2t^2 - 2t - 7)e^{2t}$$

   a. Find points $(x, y)$ on the parametric curve where the tangent lines are vertical.
   b. Find points $(x, y)$ on the parametric curve where the tangent lines are horizontal.
   c. Find the interval(s), in terms of $t$, where the parametric curve is concave upward.
   d. Find the interval(s), in terms of $t$, where the parametric curve is concave downward.

3. Find the exact length of the parametric curve

$$x = \sin^3(3\theta),\ \ y = \cos^3(3\theta),\ \ 0 \le \theta \le \frac{\pi}{6}$$

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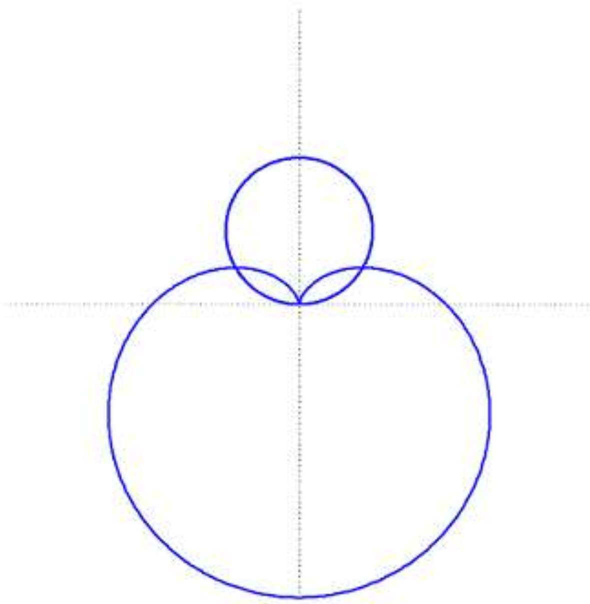
4. Given the two Cartesian points below:
    a. $(-\sqrt{11}, -5)$
    b. $(-1, \sqrt{3})$

   For each point,
    (i) Find the polar coordinates $(r, \theta)$ of the point, where $r > 0$ and $0 \leq \theta < 2\pi$.
    (ii) Find the polar coordinates $(r, \theta)$ of the point, where $r < 0$ and $0 \leq \theta < 2\pi$.

   Give exact answers.

5. Consider the polar graphs $r = 1 - \sin\theta$ and $r = \sin\theta$, shown below.



    a. Find the polar coordinates $(r, \theta)$ for all points of intersection on the figure. Hint: Not all points can be found algebraically.
    b. Find the area of the region which lies inside both curves.
    c. Find the slope of the tangent line to $r = 1 - \sin\theta$ at $\theta = -\frac{\pi}{3}$.
    d. Find the Cartesian equation of the line tangent to $r = 1 - \sin\theta$ at $\theta = -\frac{\pi}{3}$.

6. Find the length of the polar curve $r = 1 - \sin\theta$ on the interval $0 \leq \theta \leq \frac{\pi}{2}$.

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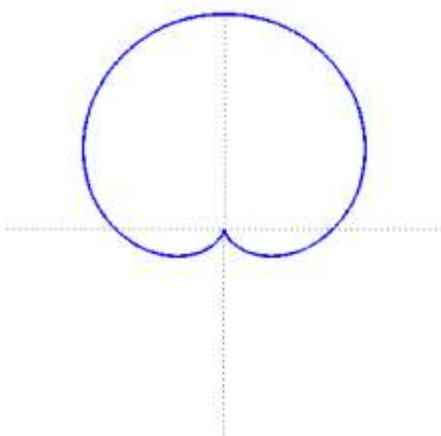
7. For some microphones, the region of sensitivity takes on the shape of a rotated cardioid

Consider the graph of $r = 1 + \sin\theta$:



Set up and evaluate the integral that represents the surface area of rotation of this cardioid about the $y$-axis. Hint: Rotate only the right half of the cardiod, and use what you know about other surface area integrals to construct this integral.