Math 2414 Graded Homework 1 (Section 7.8, 8.1, 8.2) Fall 2019

Instructions:

1) **Due: During the first 10 minutes of your problem session the week of Sept. 23, 2019**
2) **Assignments must be submitted in a blue book – no exceptions. Bluebooks are available at the on campus/off campus bookstore, student union, etc.**
3) **On the front cover, write the following at the top:**
   a) **TA name**
   b) **Your name**
   c) **Assignment name**
   d) **Problem session number**
4) **You may use both sides of each page, but start a new problem at the top of a new side.**
5) **Solutions must be presented neatly, completely, and with logical flow.**
6) **15% will be deducted for assignments turned in after the first 10 minutes of class.**
7) **25% will be deducted for assignments which are not neat and orderly.**
8) **15% will be deducted for assignments without your TA's name.**
9) **Assignments will not be accepted after class.**

Section 7.8

1) Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

   a) $\int_9^{25} \frac{x}{\sqrt{x-9}}\,dx$

   b) $\int_{\frac{\pi}{2}}^{\pi} \frac{\cos^3 x}{\sin x}\,dx$

   c) $\int_{-\infty}^{\infty} \frac{e^x}{e^{2x}+1}\,dx$

   d) $\int_0^{\infty} \frac{1}{x^2+3x+2}\,dx$

2) Consider the integral

   $\int_0^{\infty} \frac{2}{x^2-9}\,dx$

   a) Rewrite this integral as the combination of improper integrals of Type 1 and Type 2 necessary to evaluate the convergence of the integral. But do not evaluate them.

   b) Evaluate the indefinite integral $\int \frac{2}{x^2-9}\,dx$

c) Determine whether the integral $\int_0^\infty \frac{2}{x^2-9}\,dx$ converges or diverges.

d) It is only necessary to evaluate one of the Type 1 or Type 2 indefinite integrals from part (a) to determine the result in part (c). Indicate which one and explain why.

3) Consider the following function

$F(s) = \int_0^\infty f(t)e^{-st}\,dt$

where s is treated as a constant inside the integral. Find $F(s)$ for $s > 0$ if $f(t) = t$.

4) Use the Comparison Test for Improper Integrals to determine whether the following integral converges or diverge.

$\int_1^\infty \frac{x\sin^2 x}{x^3+4}\,dx$

Section 8.1

1) Find the exact length of the curve $y = \frac{x^2}{4} - \frac{\ln(5x)}{2}$ on the interval $1 \le x \le 3$.

2) Consider the arc length of the curve $x = \ln y$ on the interval $1 \le y \le 2$.
a) Set up, but do not solve, the integral for the exact length of this curve with respect to y.
b) Use a trigonometric substitution to find the exact arc length.

Section 8.2

1) Consider the portion of a hyperbola $y = \sqrt{x^2-1}$, on $\sqrt{5} \le x \le \sqrt{10}$.
a) Set up, but DO NOT evaluate, the integral with respect to x that represents the surface area obtained by roating the curve about the y-axis.
b) Set up AND EVALUATE the integral with respect to y which represents the area of the surfave obtained by rating the curve about the y-axis.

2) Consider the curve $y = e^{2x-1}$ on $1 \le y \le e$ (Note that the range is given in terms of y.
Set up, but DO NOT evaluate the integral representing the area of the surface obtained by rotating the curve about the y-axis.