1

Recall: ~~Review~~

1. Row picture for $Ax=b$

   2-D case: two lines meet at a point

   3-D case: 3 planes meet at a point

2. Column picture for $Ax=b$

   combination of columns of A gives b

(3. Multiplication by columns.

   Ax: a combination of columns of A)

§2.2 The idea of elimination (Gaussian elimination)

2-D case

multiplier ['mʌltɪ,plaɪə]n. 乘子.

$$\begin{cases} 1x - 2y = 1 & ① \\ 3x+2y = 11 & ② \end{cases} \xrightarrow{?} \begin{cases} x - 2y = 1 \\ \quad 8y = 8 \end{cases} \underset{\text{back substitution}}{\Rightarrow} \begin{cases} x = 3 \\ y = 1 \end{cases}$$

(Goal: produce an upper triangular system via elimination)

~~② - 3①~~ eliminate x: ② $-$ 3①

$8y = 8$ ③

$$\begin{cases} 1x - 2y = 1 & ① \\ \quad 8y = 8 & ③ \end{cases}$$

Pivot: first nonzero in the row that does the elimination.

Multiplier: $\dfrac{\text{entry to eliminate}}{\text{pivot}}$

Note: 1. Pivot is nonzero

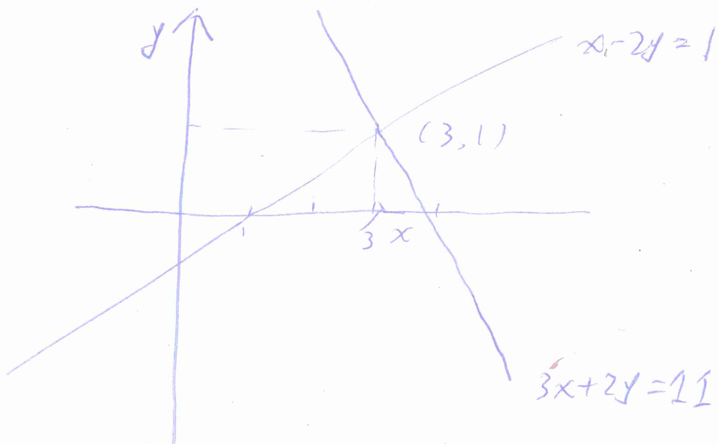
2. Pivots are on the diagonal of the triangle (after elimination)

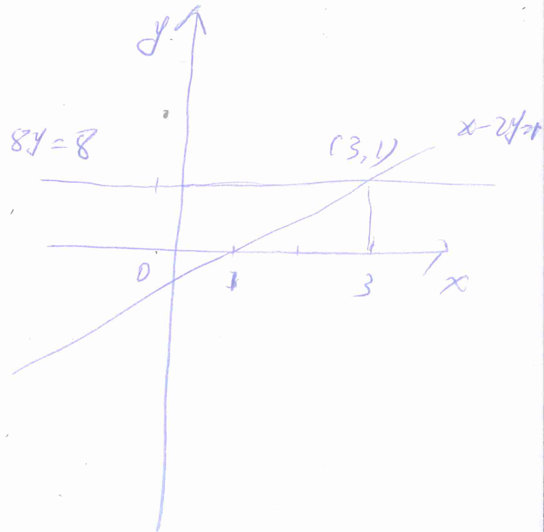
1st Pivot

$$\begin{cases} 4x - 8y = 4 & ④ \\ 3x + 2y = 11 & ⑤ \end{cases} \xrightarrow{⑤ - \frac{3}{4}④} \begin{cases} 4x - 8y = 4 \\ \quad 8y = 8 \end{cases}$$

2nd Pivot

(2)



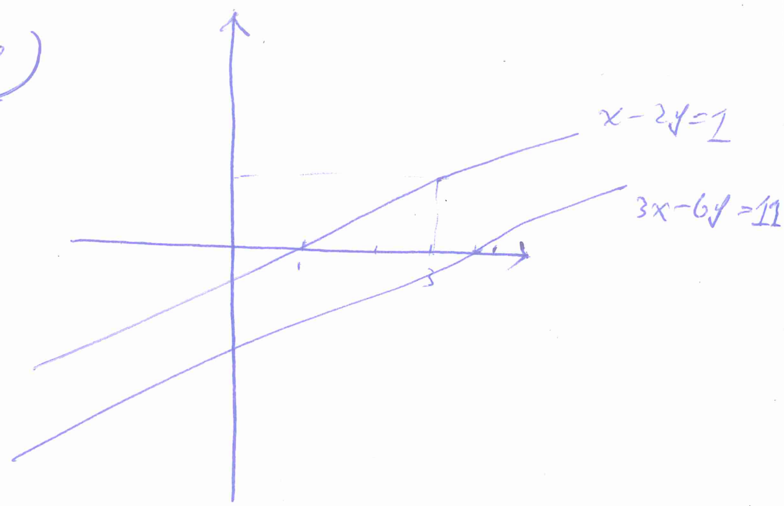


Breakdown of Elimination (pivot = zero)

Example 1

$$\begin{cases} x - 2y = 1 & (1) \\ 3x - 6y = 11 & (2) \end{cases} \xrightarrow{(2) - \frac{3}{1}(1)} \begin{cases} x - 2y = 1 \\ 0y = 8 \end{cases}$$ no solution!

(The second pivot position is zero
Zero is never allowed as a pivot)



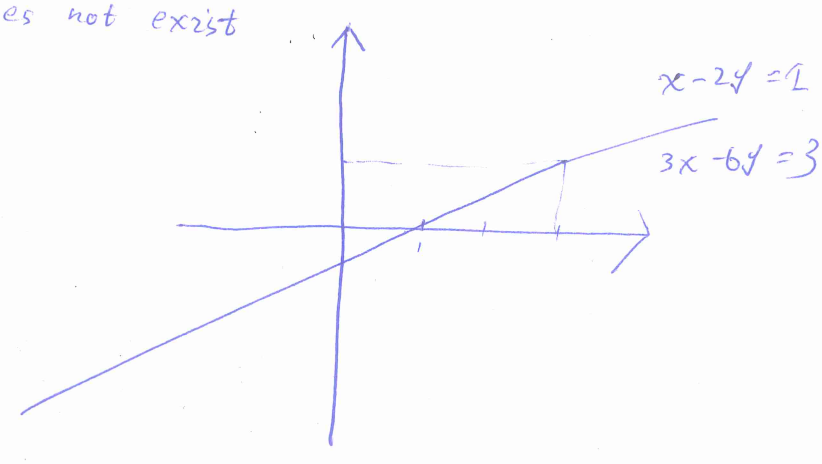
Example 2.

$$\begin{cases} x - 2y = 1 & (1) \\ 3x - 6y = 3 & (2) \end{cases} \xrightarrow{(2) - \frac{3}{1}(1)} \begin{cases} x - 2y = 1 \\ 0y = 0 \end{cases}$$

1° ~~no pivo~~ second pivot does not exist

2° y can be ~~a~~ any number

3° $x = 1 + 2y$



Failure: For n equations we do not get n pivots
Elimination lead to an equation $0y = 8$ (no solution)
or $0y = 0$ (many solutions)

(Success comes with n pivots. But we may need to exchange the n equations)

Example 3

$$\begin{cases} 0x + 2y = 4 \\ 3x - 2y = 5 \end{cases} \xrightarrow{\text{permutation}} \begin{cases} 3x - 2y = 5 \\ 2y = 4 \end{cases}$$

$\downarrow$ back substitution.

$$\begin{cases} x = 3 \\ y = 2 \end{cases}$$

3-D case

$$\begin{cases} 2x + 4y - 2z = 2 & (1) \\ 4x + 9y - 3z = 8 & (2) \\ -2x - 3y + 7z = 10 & (3) \end{cases} \xrightarrow[(3) - \frac{(-2)}{2}(1)]{(2) - \frac{4}{2}(1)} \begin{cases} 2x + 4y - 2z = 2 & (1) \\ y + z = 4 & (4) \\ y + 5z = 12 & (5) \end{cases}$$

$$\xrightarrow{(5) - \frac{1}{1}(4)} \begin{cases} 2x + 4y - 2z = 2 \\ y + z = 4 \\ 4z = 8 \end{cases} \xrightarrow{\text{back subs}} \begin{cases} x = -1 \\ y = 2 \\ z = 2 \end{cases}$$

$$Ax = b \xrightarrow{\text{Elimination}} \underset{\substack{\uparrow \\ \text{upper triangular matrix}}}{U}x = c \xrightarrow{\text{back subs}} x = U^{-1}c$$

Step 1. Use equation 1 to create zeros below first pivot
Step 2 Use new equation 2 to ——— second ———
Step 3 keep going to find n pivots and U.

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Review:

1. $Ax = b \xrightarrow{\text{elimination}} Ux = c \xrightarrow{\text{back subs}} x = U^{-1}c$

2. pivot can not be zero. ~~multiplier = $\frac{\text{entry to eliminate}}{\text{pivot}}$~~

multiplier $l_{ij} = \dfrac{\text{entry to eliminate in row } i}{\text{pivot in row } j}$

3. If zero is in pivot position, exchange rows if there is a nonzero below it.

4. When breakdown happens, $Ax=b$ has no solution or many.