Linear Algebra

What is linear algebra?

website: www.utdallas.edu/~yxz180064/algebraspring/algebra.html

- algebraic aspect: solving linear equations (fundamental problem)

  least squares and so on.

$$\begin{cases} 2x - y = 0 \\ -x + 2y = 3 \end{cases} \Rightarrow \begin{cases} x = ? \\ y = ? \end{cases}$$

- geometric aspect: vector spaces, linear transformations etc.
- applications: engineering, statistics, physics etc.

Chapter 1 Introduction to Vectors

Outline: basic ~~operators~~ operations for vectors

how to study: $\underbrace{\text{2-D} \longrightarrow \text{3-D}}_{\text{geometry}} \longrightarrow \text{n-D}$

$\downarrow$ algebra

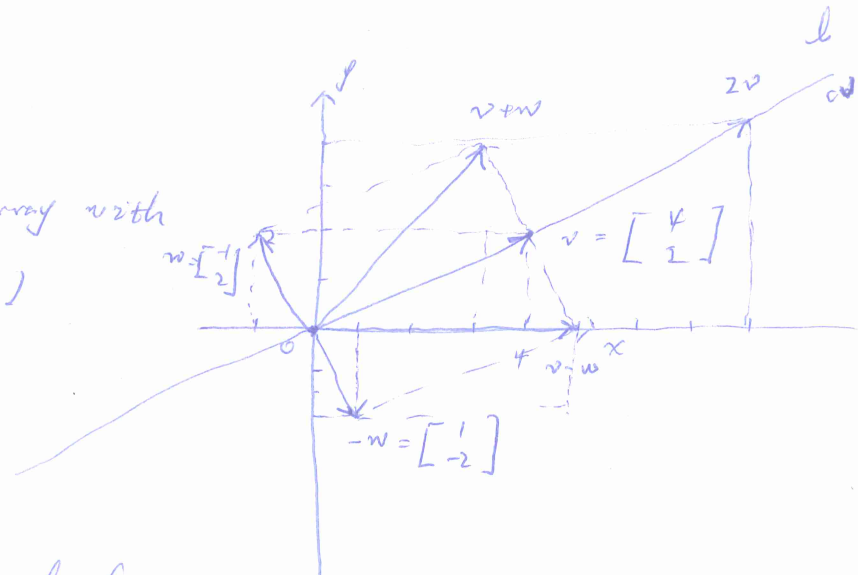
§1.1 Vectors and Linear Combinations.

2-D vectors (analytic geometry)

geometry: an ~~array~~ arrow (in the plane)

algebra: $v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ (a column array with 2 components)

ex: $v_1 = 4$, $v_2 = 2$



vector addition:

$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$, $w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$

$v + w = \begin{bmatrix} v_1 + w_1 \\ v_2 + w_2 \end{bmatrix}$ (geometry: diagonal of a parallelogram)

$$v + w = \begin{bmatrix} 4 \\ 2 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

scalar multiplication:

$cv = \begin{bmatrix} cv_1 \\ cv_2 \end{bmatrix}$ (geometry: expand and contract of an arrow)

ex: $c = 2$, ✓

$c = -1$, ✓

$c = 0$, $\mathbf{0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Linear Combinations

Let v and w be two vectors

a linear combination of v and w:

$cv + dw$, for some numbers c and d

ex: $1v + 1w =$

$1v - w =$

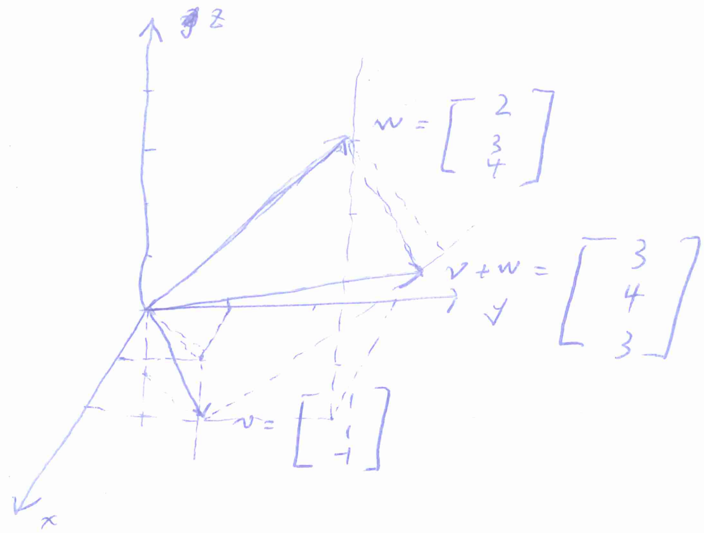
$0v + 0w = 0$ (the origin)

$cv + 0w = ?$ for any c

3-D vectors

geometry: an arrow (in 3-D space)

algebra: $v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$



ex: $v = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

vector addition:

$v = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ and $w = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$

$$v + w = \begin{bmatrix} 1+2 \\ 1+3 \\ -1+4 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 3 \end{bmatrix}$$

~~vector~~ scalar ~~vector~~ multiplication

$v = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

$cv = \begin{bmatrix} c \\ c \\ -c \end{bmatrix}$

Note: $v = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ is also written as $v = (1, 1, -1)$ (reason: save space)

linear combination

$v = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, $w = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$,

$cv + dw = \begin{bmatrix} c+2d \\ c+3d \\ -c+4d \end{bmatrix}$ $\quad v + 4w = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 8 \\ 12 \\ 16 \end{bmatrix} = \begin{bmatrix} 9 \\ 13 \\ 15 \end{bmatrix}$.

③ Note: linear combination in n-D space ?

$$v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}, \quad w = \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}$$

$$v + w = \begin{bmatrix} v_1 + w_1 \\ \vdots \\ v_n + w_n \end{bmatrix}, \qquad cv = \begin{bmatrix} cv_1 \\ \vdots \\ cv_n \end{bmatrix}$$

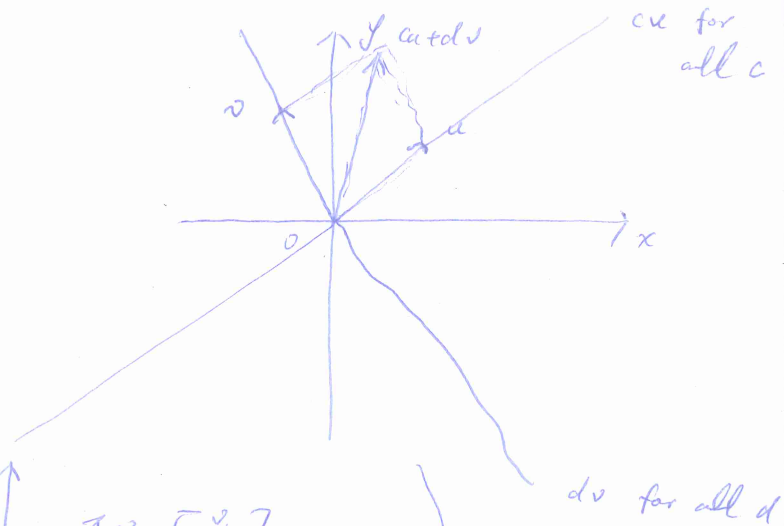
The important questions.

Let u, v, w be vectors in 3-D space.

1. What is the picture of <u>all</u> combinations cu ?
2. ______________________ $cu + dv$?
3. ______________________ $cu + dv + ew$?

Answer: ~~Tx~~ Typically:

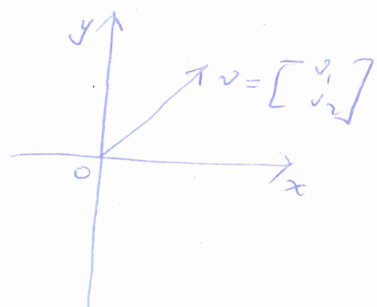
1. a line through $(0,0,0)$;
2. a ~~plane~~ plane ___ ___ ___;
3. the whole 3-D space.



Review:

1. a vector in 2-D space:

$$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$



2. ~~$v_1 + v_2$~~ $v + w = (v_1 + w_1, v_2 + w_2)$

   $cv = (cv_1, cv_2)$

3. a linear combination of u, v and w is $cu + dv + ew$.
4. three important questions.

Example Let $v = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $w = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, $b = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$.

Find c, d such that $cv + dw = b$

$$c\begin{bmatrix} 2 \\ -1 \end{bmatrix} + d\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

④

$$\begin{cases} 2c - d = 0 \\ -c + 2d = 3 \end{cases} \Rightarrow \begin{cases} c = 1 \\ d = 2 \end{cases}$$