MATH 2418: Linear Algebra

Assignment 9 (sections 4.1, 4.2, 4.3 and 4.4)

Due: `April 10, 2019` Term: `Spring, 2019`

[First Name] [Last Name] [Net ID]

Suggested problems (do not turn in):Section 4.1: 1, 2, 3, 5, 6, 7, 10, 11, 14, 16 20, 21, 22, 24, 25, 26, 28 29; Section 4.2: 1, 2, 3, 4, 5, 6, 7, 8, 11, 13, 16, 17, 21, 23, 24; Section 4.3: 1, 2, 3, 4, 5, 7, 9, 10, 12, 13, 14, 17, 18, 19, 21, 22; Section 4.4: 1, 3, 4, 5, 6, 9, 13, 15, 18, 19, 22, 24. Note that solutions to these suggested problems are available at *math.mit.edu/linearalgebra*

1. [10 points] Find a vector orthogonal to the null space of matrix A, where

   (a) (5 pts) $A = \begin{bmatrix} 1 \\ 2 \\ -3 \\ 4 \end{bmatrix} \begin{bmatrix} -2 & 3 & -3 \end{bmatrix}$.

   (b) (5 pts) $A = \begin{bmatrix} 2 & 4 & 6 & 8 \\ -3 & -6 & -9 & -12 \\ 3 & 6 & 9 & 12 \end{bmatrix}$

2. [10 points] Find a basis for the orthogonal complement to the row space of the matrix

$$\begin{bmatrix} 2 & -1 & 3 & 4 & -5 & 6 \\ 6 & -3 & -8 & 12 & -15 & 18 \\ 4 & -2 & 0 & 8 & -10 & 6 \\ 4 & -2 & 0 & 8 & -10 & 12 \end{bmatrix}$$

3. [10 points] Let P be the set of points $(x, y, z) \in \mathbb{R}^3$ satisfying the equation $-2x + y - 3z = 0$. Find a unit vector $\mathbf{n}$ orthogonal to P.

4. [10 points] For the projection onto the vector $\mathbf{a} = \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}$

(a) (3 points) Find the projection matrix P.

(b) (3 points) Project the vectors $\mathbf{b}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\mathbf{b}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

(c) (1 point) Find the errors $\mathbf{e}_1 = \mathbf{b}_1 - P\mathbf{b}_1$ and $\mathbf{e}_2 = \mathbf{b}_2 - P\mathbf{b}_2$

5. [10 points] Find the minimal distance from the point $\mathbf{b} = \begin{bmatrix} 1 \\ -3 \\ 1 \\ 5 \end{bmatrix}$ to the space of all linear combinations of the vectors $\mathbf{a}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$ and $\mathbf{a}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$

6. [10 points] Find the best line $q(t) = At+B$ approximating the data set b at the times $t = 0, 1, 2, 3, 4$.

(a) (5 points) $b = -1, -1, 2, 0, 0$

(b) (5 points) $b = -1, 0, 2, 0, 1$

7. [10 points] Find the closest parabola $q(t) = At^2 + Bt + C$ approximating the data set

time	t	-1	0	1	2	3
data	q	5	2	1	2	5

8. [10 points] Let $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\mathbf{e}_2 = \begin{bmatrix} 0 \\ -3/5 \\ 4/5 \end{bmatrix}$.

(a) (5 points) Find vector $\mathbf{e}_3$ such that $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ form an orthonormal basis in $\mathbb{R}^3$.

(b) (1 point)In how many different ways can you choose $\mathbf{e}_3$?

(c) (4 points) Express vector $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ as a linear combination of vectors $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$.

9. [10 points] Find orthogonal vectors $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ by Gram-Schmidt from the vectors $\mathbf{a} = (1, -2, 2, 0)$, $\mathbf{b} = (-2, 2, 0, 1)$ and $\mathbf{c} = (-2, 0, 1, 2)$.

10. [10 points] True or False? Circle your answer and **provide a justification** for your choice.

(a) **T F:** Row space and column space of the square matrix coincide.

(b) **T F:** If vector **a** belongs to the null-space of some matrix A then any **b** orthogonal to **a** belongs to the row space of A.

(c) **T F:** If vectors **b** and **a** are orthogonal then projection of **b** onto line through **a** has no errors.

(d) **T F:** If matrix A is a square matrix then $A(A^TA)^{-1}A^T = I$.

(e) **T F:** Least square approximation finds the line passing through all points in the data set.

(f) **T F:** If P is a projection matrix then $P^3 = P$.

(g) **T F:** Projection of the vector onto the subspace minimizes the length of the error vector.

(h) **T F:** If Q is square orthogonal matrix such that $Q^{2019} = I$ then $Q^{2018} = Q^T$.

(i) **T F:** If three vectors in $\mathbb{R}^3$ are orthonormal then they form a basis in $\mathbb{R}^3$.

(j) **T F:** If columns of matrix A are orthogonal then rows of A are independent.