MATH 2418: Linear Algebra

Assignment 8 (sections 3.5 and 4.1)

Due: `March 27, 2019` Term: `Spring, 2019`

[First Name] [Last Name] [Net ID]

Suggested problems (do not turn in):Section 3.5: 1, 2, 4, 5, 6, 9, 12, 13, 14, 18, 19, 22, 23, 26, 27; Section 4.1: 1, 2, 3, 5, 6, 7, 10, 11, 14, 16 20, 21, 22, 24, 25, 26, 28 29. Note that solutions to these suggested problems are available at *math.mit.edu/linearalgebra*

1. [10 points] Let V be a subspace of $\mathbb{R}^3$, spanned by the vectors $(2, 3, 1)$ and $(0, 4, 4)$.

   (a) (4 pts) Construct a matrix A, such that $R(A) = C(A^T) = V$.

   (b) (4 pts) Construct a matrix B such that $N(B) = V$.

   (c) (2 pts) Find AB.

2. [10 points] Find the dimensions and bases for the four fundamental subspaces of matrix

$$A = \begin{bmatrix} 2 & -3 & 4 & 5 & -6 \\ 4 & -5 & 8 & 9 & -14 \\ 6 & -10 & 12 & 16 & -16 \end{bmatrix}$$

3. [10 points] Let $\mathbf{u} = [2, 3, 1]$, $\mathbf{v} = [0, 1, -1]$, $\mathbf{a} = [2, 1]$ and $\mathbf{b} = [-3, -2]$.

(a) (3 pts) Find the rank of the matrix $B = \mathbf{a}^T\mathbf{u} + \mathbf{b}^T\mathbf{v}$.

(b) (3 pts) Find the dimension of $N(B)$.

(c) (4 pts) Find the basis for $C(B)$.

4. [10 points] Construct a (4×4) matrix whose column space equals to its null-space, or explain why it is impossible.

5. [10 points] Find the basis for the $C(A)$ and $R(A) = C(A^T)$ where $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & 6 & 7 & 8 \\ 0 & 0 & 9 & 1 \end{bmatrix}$.

6. [10 points] Find a vector orthogonal to the null space of matrix A, where

(a) (5 pts) $A = \begin{bmatrix} 1 \\ 2 \\ -3 \\ 4 \end{bmatrix} \begin{bmatrix} -2 & 3 & -3 \end{bmatrix}.$

(b) (5 pts) $A = \begin{bmatrix} 2 & 4 & 6 & 8 \\ -3 & -6 & -9 & -12 \\ 3 & 6 & 9 & 12 \end{bmatrix}$

7. [10 points]

(a) (5 pts) Find a vector $\mathbf{x} = [x_1, x_2, x_3]$ such that $\mathbf{x}^T \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 1$ and $\mathbf{x}$ is orthogonal to $\begin{bmatrix} 1 \\ -2 \\ 2 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 1 \\ 0 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 0 \\ -1 \\ -2 \end{bmatrix}$.

(b) (5 pts) Does there exist a solution to the system

$$\begin{cases} \begin{array}{rrrl} x & +2y & +3z & = 1 \\ -2x & +y & & = 0 \\ 2x & & -z & = 0 \\ -x & -3y & -2z & = 0 \end{array} \end{cases}$$

8. [10 points] Find a basis for the orthogonal complement to the null-space of the matrix

$$\begin{bmatrix} 2 & -1 & 3 & 4 & -5 & 6 \\ 6 & -3 & -8 & 12 & -15 & 18 \\ 4 & -2 & 0 & 8 & -10 & 6 \\ 4 & -2 & 0 & 8 & -10 & 12 \end{bmatrix}$$

9. [10 points] Let P be the set of points $(x, y, z) \in \mathbb{R}^3$ satisfying the equation $-2x + y - 3z = 0$. Find a unit vector $\mathbf{n}$ orthogonal to P.

10. [10 points] True or False? Circle your answer and **provide a justification** for your choice.

(a) **T F:** If the row space of matrix A equals to its column space then $A^T = A$.

(b) **T F:** If A is an invertible $(m \times m)$ matrix and B is $(n \times m)$ matrix, then A and BA have the same null-space.

(c) **T F:** Dimensions of the column-spaces for the matrices A and $\begin{bmatrix} A & A & A \\ A & A & A \end{bmatrix}$ coincide.

(d) **T F:** Let $B = AA^T$. Then column space and row space of B coincide.

(e) **T F:** If null-space and left null-space of matrix A coincide, then $A = A^T$.