MATH 2418: Linear Algebra

Assignment 7 (sections 3.3 and 3.4)

Due: `March 13, 2019` Term: `Spring, 2019`

[First Name] [Last Name] [Net ID]

Suggested problems (do not turn in): Section 3.3: 1, 2, 3, 4, 5, 7, 16, 17, 25, 26, 27, 28, 29; Section 3.4: 1, 2, 3, 4, 5, 11, 12, 13, 15, 16, 17, 18, 24, 26, 27, 35. Note that solutions to these suggested problems are available at *math.mit.edu/linearalgebra*

1. [10 points] Given linear system $\begin{cases} x_2 + x_3 + 3x_4 + x_5 = 0 \\ 2x_1 + 3x_2 + x_3 + \; x_4 \qquad = -1 \\ 6x_1 + 2x_2 \qquad + 6x_4 + x_5 = 1 \end{cases}$ corresponding to $A\mathbf{x} = \mathbf{b}$.

   (a) Solve the system.

   (b) Write your solution as $\mathbf{x} = \mathbf{x}_p + \mathbf{x}_n$, where $\mathbf{x}_p$ is the particular solution of $A\mathbf{x} = \mathbf{b}$ and $\mathbf{x}_n$ is a linear combination of special solutions of $A\mathbf{x} = \mathbf{0}$.

   (c) What is the rank of the coefficient matrix A?

2. [10 points] Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 3 \\ 4 & 5 & 3 \end{bmatrix}$ and $U = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$. Which of the spaces $C(A)$, $C(U)$, $C(A^T)$, $C(U^T)$ are the same?

3. [10 points] Find the LU factorization of A and the complete solution to $A\mathbf{x} = \mathbf{b}$.

$$A = \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 1 \\ -1 & 1 & 0 & 2 \end{bmatrix}, \; b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

4. [10 points] Determine if the given vectors form a basis for the vector space specified.

(a) $V = M_{22}$ (the vector space of all 2×2 matrices), given set of vectors

$$\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}.$$

(b) $V = M_{22}$ (the vector space of all 2×2 matrices), given set of vectors

$$\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

(c) $V = P_2$(the vector space of all polynomials of degree ≤ 2), given set of vectors

$$4 + x,\ x^2 - x - 1,\ x^2 + x - 3.$$

5. [10 points] Let $\mathbf{v}_1 = (-2, -7, 2)$, $\mathbf{v}_2 = (5, 1, -5)$, $\mathbf{v}_3 = (-4, -9, 4)$, $\mathbf{v}_4 = (-3, -6, 3)$, $\mathbf{v}_5 = (1, 2, -1)$. Find a subset of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5\}$ that forms a basis for the subspace of $\mathbb{R}^3$ spanned by those five vectors. Express each non-basis vector as a linear combination of basis vectors.

6. [10 points] Let $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ be three linearly independent vectors in $\mathbb{R}^3$.

(a) Find the rank of the matrix $A = \begin{bmatrix}(\mathbf{v}_1 - \mathbf{v}_2) & (\mathbf{v}_2 - \mathbf{v}_3) & (\mathbf{v}_3 - \mathbf{v}_1)\end{bmatrix}$.

(b) Find the rank of the matrix $B = \begin{bmatrix}(\mathbf{v}_1 + \mathbf{v}_2) & (\mathbf{v}_2 + \mathbf{v}_3) & (\mathbf{v}_3 + \mathbf{v}_1)\end{bmatrix}$.

7. [10 points] Prove that $B = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \right\}$ is a basis for $U_{2\times 2}$, the vector space of all 2×2 upper triangular real matrices.

8. [10 points] Find a basis for the subspace of $\mathbb{R}^3$ spanned by the vectors $\mathbf{v_1} = (0, 2, 6)$, $\mathbf{v_2} = (1, 3, 2)$, $\mathbf{v_3} = (4, 1, 4)$, $\mathbf{v_4} = (3, 1, 6)$, $\mathbf{v_5} = (1, 0, 1)$. Express each non-basis vector as a linear combination of basis vectors.

9. [10 points] (a) Determine if the vectors $2 - x^2,\ 1 + x,\ 1 + 2x$ form a basis for P_2(the vector space of all polynomials of degree ≤ 2).

(b) Determine if the vectors $(1, 3, 2),\ (0, 2, 6),\ (4, 1, 4)$ form a basis for $\mathbb{R}^3$.

10. [10 points] Consider the plane P represented by the equation $2x + 3y - 2z = 0$.

(a) Find a basis for P.

(b) Find a basis for the intersection of P with $yz-$plane.