MATH 2418: Linear Algebra

Assignment 6 (sections 3.1 and 3.2)

Due: `March 06, 2019` Term: `Spring, 2019`

[First Name] [Last Name] [Net ID]

Suggested problems (do not turn in): Section 3.1: 1, 2, 5, 9, 10, 11, 12, 19, 20, 24, 26; Section 3.2: 1,2, 3, 4, 8, 12, 15, 18, 31. Note that solutions to these suggested problems are available at *math.mit.edu/linearalgebra*

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1. [10 points] Find the nullspace of $A = \begin{bmatrix} 0 & 1 & 1 & 3 & 1 \\ 2 & 3 & 1 & 1 & 0 \\ 6 & 2 & 0 & 6 & 1 \end{bmatrix}$. What is rank of A? Also find the special solutions of $A\mathbf{x} = \mathbf{0}$.

2. [10 points] (a) Suppose matrix A reduces into echelon form U, prove that $N(A) = N(U)$.

(b) Write a 2×2 matrix A such that $C(A) \neq C(U)$, where U is the echelon form of matrix A.

3. [10 points] (a) Determine if the vectors $\mathbf{v}_1 = (1, 2, -1)$, $\mathbf{v}_2 = (3, 8, 0)$, $\mathbf{v}_3 = (1, 1, 1)$ span $\mathbb{R}^3$.

(b) Determine if $\mathbf{b} = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}$ is in the column space of $A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 8 & 1 \\ -1 & 0 & 1 \end{bmatrix}$. If yes, write $\mathbf{b}$ as a linear combination of columns of A.

4. [10 points] Determine if the set consisting of

(a) (2 pts) all $(x, y, z) \in \mathbb{R}^3$ with $x = -z$ is a subspace of $\mathbb{R}^3$

(b) (2 pts) all $(x, y, z) \in \mathbb{R}^3$ with $x = -z - 2$ is a subspace of $\mathbb{R}^3$

(c) (3 pts) all vectors $\mathbf{x} \in \mathbb{R}^n$ satisfying $A\mathbf{x} = \mathbf{0}$ where A is an $n \times n$ real matrix, is a subspace of $\mathbb{R}^n$.

(d) (3 pts) $D_{2\times 2} = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : a, b \in \mathbb{R} \right\}$ is a subspace of $M_{2\times 2}$, the vector space of all 2×2 real matrices.

5. [10 points] Determine if column space of the matrix $A = \begin{bmatrix} 2 & 2 & 2 & 1 & 1 & 1 \\ 0 & 0 & 0 & 9 & 9 & 8 \\ 1 & 1 & 1 & 6 & 7 & 1 \\ 7 & 8 & 9 & 9 & 0 & 2 \end{bmatrix}$ contains the vector $\mathbf{b} = \begin{bmatrix} 1 \\ 7 \\ 7 \\ 6 \end{bmatrix}$.

6. [10 points] Find reduced row echelon form of the matrix $A = \begin{bmatrix} 1 & 2 & -3 & 4 & -5 \\ 2 & 3 & -6 & 7 & 9 \\ -2 & 3 & 6 & -8 & 10 \\ 1 & 2 & -3 & 4 & 6 \end{bmatrix}$. Which variables are free?

7. [10 points] Given $A = \begin{bmatrix} 1 & -2 & 3 & -2 & -1 & 1 \\ 2 & -4 & 6 & -1 & 1 & 3 \\ 3 & -6 & 9 & -1 & 2 & 1 \\ -4 & 8 & -12 & 2 & -2 & -3 \end{bmatrix}$

(a) Find the nullspace $N(A)$.

(b) Find three special solutions of $A\mathbf{x} = \mathbf{0}$.

(c) What is the rank of A?

8. [10 points] Is the vector $\begin{bmatrix} 3 \\ -1 \\ 3 \end{bmatrix}$ a linear combination of $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 8 \end{bmatrix}$? Explain your answer.

9. [10 points] Answer the followings(you do not need to show your work).

(a) Write 1×3 matrix A whose null space is the plane $4x - 5y + 6z = 0$

(b) Write down a matrix A such that $N(A)$ is the set of all linear combinations of $(2, 0, 1, 7)$ and $(2, 0, 1, 8)$

(c) Construct a matrix A whose column space contains $(-3, 0, 3)$ and $(1, 1, 1)$ and the nullspace contains $(1, 2, 3)$.

(d) Construct a 2×2 matrix whose null space equals to its column space.

10. [10 points] True or False? Circle your answer and **provide a justification** for your choice.

(a) **T F:** Intersection of two planes in $\mathbb{R}^3$ is a subspace in $\mathbb{R}^3$.

(b) **T F:** Set of all singular 2×2 matrices form a subspace in M_{22}.

(c) **T F:** An invertible matrix has no free variables.

(d) **T F:** Planes $2x + 3y - z = 2018$ and $-4x - 6y + 2z = 1$ are parallel.

(e) **T F:** Matrices $\begin{bmatrix} 1 & 2 & 3 \\ -2 & -4 & -5 \\ -1 & -2 & -4 \end{bmatrix}$ and $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ have the same null space.