MATH 2418: Linear Algebra

Assignment 5 (sections 2.5, 2.6, 2.7, 3.1)

Due: `February 27, 2019` Term: `Spring, 2019`

[First Name] [Last Name] [Net ID]

Suggested problems (do not turn in):
Section 2.5: 1, 5, 6, 7, 10, 11, 12, 13, 18, 22, 25, 27, 29, 44.
Section 2.6: 1, 3, 4, 6, 8, 9, 10, 13, 14, 17, 22, 23.
Section 2.7: 1,2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 20, 21, 22, 23.
Section 3.1: 1, 2, 5, 9, 10, 11,12, 19, 20, 24, 26.
Note that solutions to these suggested problems are available at *math.mit.edu/linearalgebra*

1. [10 points] Use the Gauss-Jordan method to find the inverse of $A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{bmatrix}$.

2. [10 points] Consider $A = \begin{bmatrix} 1 & 3 \\ 2 & 8 \end{bmatrix}$.

(a) (2 points). Use elementary row operations to reduce A in to I.

(b) (2 points). List all corresponding elementary matrices.

(c) (3 points). Find A^{-1} as a product of elementary matrices.

(d) (3 points). Express A as a product of elementary matrices.

3. [10 points] Solve the following problems and justify your answers by showing your work.

   (a) (2 points). Give example of 2×2 non-zero matrices A, B, C such that $AB = AC$ but $B \neq C$.

   (b) (2 points). Write 2×2 invertible matrices A and B such that $A + B$ is not invertible.

   (c) (2 points). Write 3×3 singular matrices A and B such that $A - B$ is non-singular.

   (d) (2 points). **T** or **F**? (Circle your answer) If A and B are invertible matrices of same size, then AB and BA are both invertible.

   (e) (2 points). **T** or **F**? (Circle your answer) If A^2 is not invertible, then A is not invertible.

4. [10 points] Find LDU decomposition of the matrix

$$A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 3 & -3 \\ -2 & -4 & 7 \end{bmatrix}$$

(a) $L =$ ____________________

(b) $D =$ ____________________

(c) $U =$ ____________________

5. [10 points] Solve the system $A\mathbf{x} = \mathbf{b}$ for

$$A = \begin{bmatrix} 1 & -1 & -1 \\ -2 & 3 & 1 \\ 3 & -7 & 2 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

using the following steps:

(a) (3 points). Compute factorization $A = LU$.

(b) (3 points). Solve $L\mathbf{y} = \mathbf{b}$ by forward substitution.

(c) (3 points). Solve $U\mathbf{x} = \mathbf{y}$ by backward substitution.

(d) (1 point). What is the solution of $A\mathbf{x} = \mathbf{b}$?

6. [10 points] Forward elimination changes $A\mathbf{x} = \mathbf{b}$ to the system $R\mathbf{x} = \mathbf{d}$. If the process of elimination subtracted 3 times row 1 from row 2 and then 5 times row 1 from row 3, what matrix connects R and $\mathbf{d}$ to the original A and $\mathbf{b}$? (That is, find E such that $R = EA$ and $E\mathbf{b} = \mathbf{d}$).

7. [10 points] Given matrix $A = \begin{bmatrix} 0 & 3 & 4 \\ 1 & 2 & 3 \\ -3 & -7 & 8 \end{bmatrix}$,

(a) (5 points). Show that A has no LU decomposition.

(b) (5 points). Find the decomposition $A = LPU$, where P is an elementary permutation matrix.

8. [10 points] Let $S = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 8 \\ 2 & 8 & 23 \end{bmatrix}$ be a symmetric matrix. Find the symmetric factorization of S as $S = LDL^{\mathbf{T}}$.

9. [10 points] (A) True or False. Circle your answer and justify it by showing your work (4 points).

(a) **T** **F**: Let A be any square matrix, then $A^{\mathbf{T}}A$, $AA^{\mathbf{T}}$, and $A + A^{\mathbf{T}}$ are all symmetric.

(b) **T** **F**: If S is invertible, then $S^{\mathbf{T}}$ is also invertible.

(c) **T** **F**: If a row exchange is required to reduce matrix A into upper triangular form U, then A can not be factored as $A = LU$.

(d) **T** **F**: Suppose A reduces to upper triangular U but U has a 0 in pivot position, then A has no LDU factorization.

(B) Solve the following problems showing your work (6 points).

(a) A symmetric matrix S reduces to $\begin{bmatrix} 3 & 9 \\ 0 & 7 \end{bmatrix}$ after performing row operations(except permutations), write the LDU decomposition of S.

Answer: $S =$ ________________________

(b) Let A and B be two symmetric matrices of same size. Which of the followings are symmetric? $A + B$, $A - B$, A^2, AB, ABA, $ABAB$, $ABABA$.

Answer: Symmetric: ____________________________

(c) Write the inverse of the permutation matrix $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$.

Answer: $P^{-1} =$ ________________.

10. [10 points] Determine if the set consisting of

(a) all $(x, y, z) \in \mathbb{R}^3$ with $x = y + z$ is a subspace of $\mathbb{R}^3$

(b) all $(x, y, z) \in \mathbb{R}^3$ with $x + z = 2018$ is a subspace of $\mathbb{R}^3$

(c) all 2×2 symmetric matrices is a subspace of M_{22}. (Here M_{22} is the vector space of all 2×2 matrices.)

(d) all polynomials of degree exactly 3 is a subspace of P_5. (Here P_5 is the vector space of all polynomials $a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$ of degree less than or equal to 5.)