MATH 2418: Linear Algebra

Assignment# 4

Due : 02/13, Wednesday Term : `Spring 2019`

[First Name] [Last Name] [Net ID]

[**Recommended Problems (do not turn in):** Sec 2.3: 1, 3, 4, 7, 8, 9, 18, 21, 25, 27, 28. Sec 2.4: 3, 6, 7, 10, 11, 12, 14, 17, 21, 26, 32]

1. Let $A = \begin{bmatrix} 1 & 5 & 2 \\ 1 & 0 & 1 \\ 3 & -2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 5 & 1 \\ 13 & -2 & 0 \end{bmatrix}$, find the products AB and BA.

2. For the matrices $A = \begin{bmatrix} 1 & 5 & 2 \\ 1 & 0 & 1 \\ 3 & -2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 5 & 1 \\ 13 & -2 & 0 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 2 \\ 3 & 2 \\ -1 & 3 \end{bmatrix}$, verify the associative property of the matrix multiplication: $(AB)C = A(BC)$.

3. Consider the 3×3 matrix $A = \begin{bmatrix} 1 & 5 & 2 \\ 1 & 0 & 1 \\ 3 & -2 & 4 \end{bmatrix}$

(a) Perform the 3 elementary row operations to reduce the matrix A into an upper triangular matrix U.

(b) Write down the elementary matrices E_{21}, E_{31}, E_{32} corresponding to the row operations in part (a).

(c) Write down a single matrix M such that $MA = U$ is upper triangular.

4. Consider the matrix A as in previous question and the vector $\mathbf{b} = (1, 2, 1)$, solve the system $A\mathbf{x} = \mathbf{b}$.

5. Given elementary matrices $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$, $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$, $E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$, and $E_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$, find the following products

(a) $E_1 E_2 =$

(b) $E_2 E_1 =$

(c) $E_3 E_4 =$

(d) $E_4 E_3 =$

6. Let $B = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ 4 & 6 & 0 \end{bmatrix}$.

(a) For any 4×3 matrix A, what is the column 3 of AB? Explain your answer.

(b) For any 3×2 matrix C, what is the row 2 of BC? Explain your answer.

(c) Is it possible to find a 3×3 matrix D such that $DB = I_3$, the 3×3 identity matrix? Explain.

(d) Is it possible to find a 3×3 matrix F such that $BF = I_3$, the 3×3 identity matrix? Explain.

7. Let $A = \begin{bmatrix} 1 & 2 & -5 \\ -1 & 0 & 1 \\ 3 & -2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 10 \\ 1 & -2 \\ 2 & 1 \end{bmatrix}$, without calculating AB, A^2 and BA, calculate the following:

(a) the entry $(AB)_{32}$ of AB

(b) the entry $(BA)_{23}$ of BA

(c) row 1 of AB

(d) column 3 of A^2.

8. Compute the following products:

(a) $\begin{bmatrix} a & 4 & 1 \\ 0 & b & 5 \\ 0 & 0 & c \end{bmatrix} \begin{bmatrix} d & 1 & 2 \\ 0 & e & 2 \\ 0 & 0 & f \end{bmatrix} =$

(b) $\begin{bmatrix} a & 0 & 0 \\ 2 & b & 0 \\ 3 & 3 & c \end{bmatrix} \begin{bmatrix} d & 0 & 0 \\ 1 & e & 0 \\ 2 & 1 & f \end{bmatrix} =$

(c) $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \begin{bmatrix} 3 & 5 & -7 \\ 2 & 4 & 1 \\ -9 & 2 & 6 \end{bmatrix} =$

(d) $\begin{bmatrix} 3 & 5 & -7 \\ 2 & 4 & 1 \\ -9 & 2 & 6 \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} =$

(e) $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} =$

9. Let $A = \begin{bmatrix} 1 & 5 & 2 \\ 1 & 0 & 1 \\ 3 & -2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 5 & 1 \\ 13 & -2 & 0 \end{bmatrix}$, find the product AB as a sum of columns of A times rows of B.

10. Answer the following(you DO NOT need to show your work).

(a) Write down two 2×2 non-zero matrices A and B such that $AB = 0$, the 2×2 zero matrix.

Answer: $A = \begin{bmatrix} & \\ & \end{bmatrix}$, $B = \begin{bmatrix} & \\ & \end{bmatrix}$

(b) Given $A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$ write down the elementary matrices E_1 and E_2 such that

$$E_1 A = \begin{bmatrix} 3 & 4 \\ 7 & 10 \end{bmatrix} \text{ and } E_2 A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Answer: $E_1 = \begin{bmatrix} & \\ & \end{bmatrix}$, $E_2 = \begin{bmatrix} & \\ & \end{bmatrix}$

(c) Given $A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$ write down the matrices D_1 and D_2 such that

$$D_1 A = \begin{bmatrix} 30 & 40 \\ -3 & -6 \end{bmatrix} \text{ and } AD_2 = \begin{bmatrix} 30 & -12 \\ 10 & -6 \end{bmatrix}$$

Answer: $D_1 = \begin{bmatrix} & \\ & \end{bmatrix}$, $D_2 = \begin{bmatrix} & \\ & \end{bmatrix}$

(d) For any two 2×2 matrices A and B, which of the following are guaranteed to be equal to $(A + B)^2$?

(a) $A^2 + 2AB + B^2$ (b) $A^2 + AB + BA + B^2$ (c) $A^2 + B^2$ (d) $A^2 - 2AB + B^2$

(e) For any two 2×2 matrices A and B, which of the following are guaranteed to be equal to $(A+B)(A-B)$?

(a) $A^2 + B^2$ (b) $A^2 - AB + BA - B^2$ (c) $A^2 - B^2$ (d) $A^2 - 2AB + B^2$