MATH 2418: Linear Algebra

Assignment# 3

Due : 02/06, Wednesday Term `Spring, 2019`

[First Name] [Last Name] [Net ID]

Recommended Text Book Problems (do not turn in): [Sec 2.1: 9, 10, 15, 16, 17, 19, 26, 29, 31, 33; Sec 2.2: 5, 6, 7, 12, 14, 15, 19, 23, 24, 25]

1. Solve the linear system $\begin{cases} x & = 1 \\ x + y & = 2 \\ & z = 2 \end{cases}$. Also, draw the column picture.

2. Given linear system $\begin{cases} x + y = 6 \\ 3x - y = 2 \end{cases}$

(a) Write down the corresponding matrix equation $A\mathbf{x} = \mathbf{b}$.

(b) Solve the system.

(c) Draw the row picture and the column picture.

(d) Write $\mathbf{b}$ as a linear combination of columns of A.

3. Consider the transformation $\mathrm{T} : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $\mathrm{T}(x, y, z) = (x + y + 3z,\ x - 2y - z)$ (i.e. T is a function that maps every vector $(x, y, z)\mathbb{R}^3$ to a vector $(x+y+3z,\ x-2y-z) \in \mathbb{R}^2$). Let $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$ be any two vectors in $\mathbb{R}^3$ and 'c' be any scalar. Prove that

   (a) $\mathrm{T}(\mathbf{u} + \mathbf{v}) = \mathrm{T}(\mathbf{u}) + \mathrm{T}(\mathbf{v})$

   (b) $\mathrm{T}(c\mathbf{u}) = c\mathrm{T}(\mathbf{u})$

4. Consider the transformation T defined in Q.N#3:

   (a) Find the 2×3 matrix A such that $\mathrm{T}(\mathbf{u}) = A\mathbf{u}$, where $\mathbf{u}$ is any vector in $\mathbb{R}^3$.

   (b) Let $\mathbf{i} = (1, 0, 0),\ \mathbf{j} = (0, 1, 0)$, and $\mathbf{k} = (0, 0, 1)$, find $\mathrm{T}(\mathbf{i}),\ \mathrm{T}(\mathbf{j})$, and $\mathrm{T}(\mathbf{k})$.

   (c) Write the 2×3 matrix P whose columns are the vectors $\mathrm{T}(\mathbf{i}),\ \mathrm{T}(\mathbf{j})$, and $\mathrm{T}(\mathbf{k})$ from part (b).

5. Find the matrix for the following linear transformations:

   (a) Rotation of vectors in $\mathbb{R}^2$ by 120° clockwise.

   (b) Maps any vector (x, y) in $\mathbb{R}^2$ to the vector $(2x + y,\ 3x - 4y)$ in $\mathbb{R}^2$.

   (c) Maps any vector (x, y) in $\mathbb{R}^2$ to give the vector $(x + y,\ 2y,\ 2x - 3y)$ in $\mathbb{R}^3$

6. Given linear system $\begin{cases} 2x + 4y - 3z = 7 \\ 8x + 17y - 3z = 39 \\ -3x - 3y + 7z = -2 \end{cases}$.

   (a) Solve by reducing into upper triangular form and back substitution.

   (b) List all multipliers used and circle all the pivots.

7. Consider the linear system $\begin{cases} ax - 3y = 6 \\ 3x + 6y = -12 \end{cases}$

(a) For what value(s) of 'a' does the elimination break down (i) permanently (ii) temporarily?

(b) Solve the system after fixing the temporary break down.

(c) Solve the system in case of permanent break down.

8. Is the system $\begin{cases} 3x + 4y - 2z = 15 \\ 9x + 13y - 3z = 26 \\ -3x - 3y + 5z = 12 \end{cases}$ solvable? If not, change the number 15 on the right side of the first equation so that the system has a solution.

9. Reduce the matrix $A = \begin{bmatrix} a & 2 & 3 \\ a & a+1 & 4 \\ a & a+1 & a \end{bmatrix}$ into upper triangular form. Determine the three values of 'a' for which the elimination fails to give three pivots.

10. True or False. Circle your answer.

(a) **T** **F**: Consider the augmented matrix $[A \quad : \mathbf{b}] = \begin{bmatrix} 4 & 5 & : b_1 \\ 0 & r & : b_2 \end{bmatrix}$. The existence and uniqueness of the solution depend on all of r, b_1, b_2.

(b) **T** **F**: If elimination fails permanently, the linear system has no solution.

(c) **T** **F**: There is only one possible linear system which reduces to $\begin{cases} x + \ \ y = 1 \\ \quad\ \ 2y = 3 \end{cases}$ after one elimination step.

(d) **T** **F**: If the equations in a linear system are multiplied by scalars, the row picture and the column picture both change.

(e) **T** **F**: The matrix that projects the vector (x, y) onto $x-$axis to produce $(x, 0)$ is $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.