MATH 2418: Linear Algebra

Assignment# 2

Due : 01/30, Wednesday   Term `Spring 2019`

[Last Name]   [First Name]   [Net ID]

Recommended Text Book Problems (do not turn in): [Sec 1.2: # 1, 2, 3, 4, 7, 8, 12, 13, 17, 31
Sec 1.3: 1, 2, 3, 5, 8, 9, 14]

---

1. Find all real values of 'm' so that angle between the vectors $\mathbf{u} = (m+1, -m+2, -3)$ and $\mathbf{v} = (-3, m+1, -m+2)$ is $120°$.

2. Given vectors $\mathbf{u} = (1, 2, -3)$ and $\mathbf{v} = (-3, 1, 2)$ in $\mathbb{R}^3$:

   (a) Calculate the dot product: $\mathbf{u} \cdot \mathbf{v}$

   (b) Find $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$

   (c) Find the angle θ between $\mathbf{u}$ and $\mathbf{v}$

   (d) Find the unit vector $\hat{\mathbf{u}}$ in the direction of $\mathbf{u}$.

   (e) Write a vector $\mathbf{a}$ of length 3 that is in the opposite direction of $\mathbf{u}$.

3. Let α, β, γ be the angles made by a vector (or a line) with positive x, y, and $z-$axis respectively. Then the numbers

$$l = \cos\alpha,\ \ m = \cos\beta,\ \ n = \cos\gamma$$

are called the **direction cosines** of the the vector (or the line).

(a) Find the direction cosines l, m, n of the vector $\mathbf{u} = (1, 2, 3)$

(b) Find the direction cosines l, m, n of the vector $\mathbf{u} = (a, b, c)$.

4. (a) Use the triangle inequality: $\|\mathbf{u}+\mathbf{v}\| \leq \|\mathbf{u}\| + \|\mathbf{v}\|$ to prove that

   (i) $\|\mathbf{u}-\mathbf{v}\| \leq \|\mathbf{u}\| + \|\mathbf{v}\|$

   (ii) $\|\mathbf{u}\| - \|\mathbf{v}\| \leq \|\mathbf{u}-\mathbf{v}\|$

(b) If $\|\mathbf{u}\| = 19$ and $\|\mathbf{v}\| = 24$, what are the smallest and largest possible values of $\|\mathbf{u}-\mathbf{v}\|$ and $\|\mathbf{v}-\mathbf{u}\|$?

5. (a) Given any two nonzero vectors $\mathbf{u}$ and $\mathbf{v}$, determine the scalar 'c' so that the vector $\mathbf{u} - c\mathbf{v}$ is perpendicular to $\mathbf{v}$.

(b) Let $\mathbf{v} = (4, 1, 3)$ and $\mathbf{u} = (1, 1, 1)$, use part (a) to find a non zero vector that is perpendicular to $\mathbf{v}$.

6. Given the 3×3 matrix $A = \begin{bmatrix} -3 & 2 & -3 \\ 2 & 3 & -8 \\ 3 & -2 & 3 \end{bmatrix}$ and the vector $\mathbf{x} = \begin{bmatrix} 3 \\ 8 \\ 1 \end{bmatrix}$ in $\mathbb{R}^3$, calculate $A\mathbf{x}$

(a) as a linear combination of columns of A

(b) with entries as dot products of rows of A and vector $\mathbf{x}$.

7. Let matrix $A = \begin{array}{c} \\ S1 \\ S2 \\ S3 \end{array} \begin{array}{c} \begin{array}{ccc} E1 & E2 & E3 \end{array} \\ \begin{bmatrix} 70 & 80 & 90 \\ 90 & 90 & 80 \\ 50 & 70 & 100 \end{bmatrix} \end{array}$ represent the Exam 1(E1), Exam 2(E2), and Exam 3(E3) scores (out of 100 points each) of 3 students S_1, S_2, and S_3. The vector $\mathbf{w} = \begin{bmatrix} 0.2 \\ 0.3 \\ 0.5 \end{bmatrix}$ represents the Exam 1, Exam 2, and Exam 3 weights ($20\%, 30\%$, and 50% respectively). Calculate and explain the meaning of $A\mathbf{w}$.

8. Given $A = \begin{bmatrix} -1 & 3 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

(a) Write the linear system corresponding to $A\mathbf{x} = \mathbf{b}$.

(b) Solve the linear system.

(c) Write your answer in the form of $\mathbf{x} = A^{-1}\mathbf{b}$. What is A^{-1}?

9. (a) Prove that the vectors $\mathbf{u} = (-1, 2, 0)$, $\mathbf{v} = (3, 1, 1)$, $\mathbf{w} = (0, 1, 1)$ are linearly independent.

(b) Prove that the vectors $\mathbf{u} = (1, 2, 1)$, $\mathbf{v} = (3, 1, 1)$, $\mathbf{w} = (5, 5, 3)$ are linearly dependent.

10. True or False. Circle your answer.

(a) **T** **F**: If the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent, then the set $\{a\mathbf{u}, b\mathbf{v}, c\mathbf{w}\}$ is also linearly independent, where a, b, c are any non zero real numbers.

(b) **T** **F**: Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ be three non zero vectors in $\mathbb{R}^2$ such that $\mathbf{u}$ is perpendicular to $\mathbf{v}$ and $\mathbf{w}$ both, then $\mathbf{v}$ and $\mathbf{w}$ must be parallel to each other.

(c) **T** **F**: Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ be three non zero vectors in $\mathbb{R}^3$ such that $\mathbf{u}$ is perpendicular to $\mathbf{v}$ and $\mathbf{w}$ both, then $\mathbf{v}$ and $\mathbf{w}$ must be parallel to each other.

(d) **T** **F**: For fixed length vectors, $\mathbf{u}$ and $\mathbf{v}$, the value of $\mathbf{u}\cdot\mathbf{v}$ is minimum when $\mathbf{u}$ and $\mathbf{v}$ are perpendicular to each other.

(e) **T** **F**: For fixed length vectors $\mathbf{u}$ and $\mathbf{v}$, the value of $\mathbf{u} \cdot \mathbf{v}$ is maximum when $\mathbf{u}$ and $\mathbf{v}$ have the same direction.