MATH 2418: Linear Algebra

Assignment 11 (Sections 5.3 and 6.1)

Due: <u>April 24th, 2019</u>   Term: <u>Spring 2019</u>

[First Name]   [Last Name]   [Net ID]

Recommended Text Book Problems (do not turn in): [Section 5.3: 1, 5, 6, 7, 14, 18, 31, 32. Section 6.1: 2, 3, 4, 9, 15, 16, 17, 24.] Solutions to these problems are available at *math.mit.edu/linearalgebra*

1. Solve the following linear system for $\mathbf{x} = (x_1, x_2, x_3)$ using Cramer's Rule.

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & -2 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

   (a) [1 point] $\begin{vmatrix} 1 & 2 & -1 \\ 0 & 3 & -2 \\ 0 & 2 & -3 \end{vmatrix} =$

   (b) [3 points] $x_1 =$

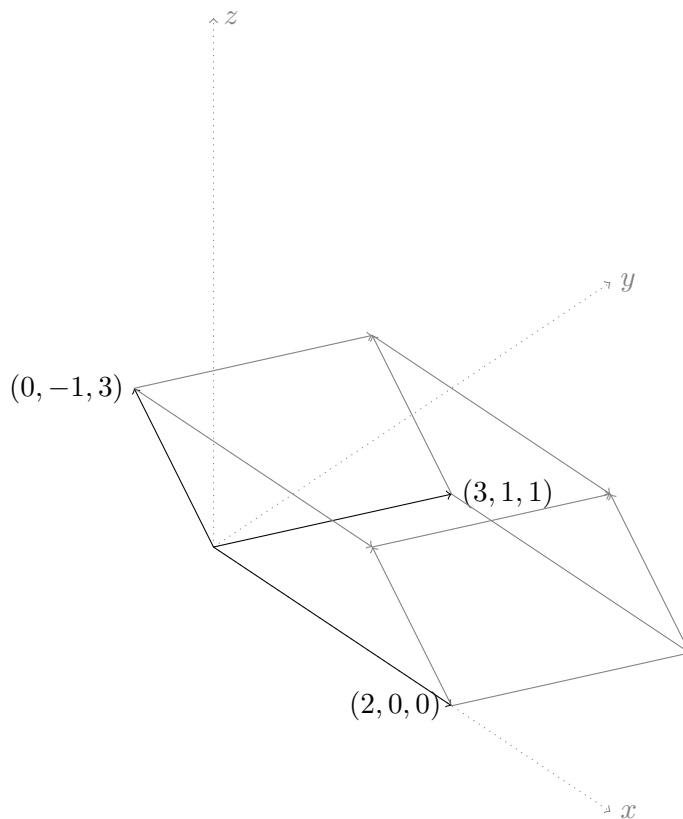   (c) [3 points] $x_2 =$

   (d) [3 points] $x_3 =$

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2. For the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 2 & -1 & 1 \end{bmatrix}$, find the determinant, cofactor matrix, and inverse.

(a) [2 points] Find $\det A$.

(b) [4 points] Find the cofactor matrix C.

(c) [4 points] Use the cofactor matrix to find A^{-1}.

3. [10 points] Find the volume of a box whose edges are the vectors $\mathbf{a}_1 = (2, 0, 0)$, $\mathbf{a}_2 = (3, 1, 1)$, and $\mathbf{a}_3 = (0, -1, 3)$.

z

y

$(0, -1, 3)$

$(3, 1, 1)$

$(2, 0, 0)$

x

4. Based on the information given below, identify an eigenvalue of the matrix A.

(a) [2 points] $A = \begin{bmatrix} .1 & .9 \\ .5 & .5 \end{bmatrix}$ and $\begin{bmatrix} .1 & .9 \\ .5 & .5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. What is an eigenvalue of A?

(b) [2 points] $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 1 \\ 3 & 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 1 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -4 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. What is an eigenvalue of A?

(c) [2 points] $A = \begin{bmatrix} 7 & 9 & 5 \\ 8 & 3 & 5 \\ -8 & 1 & -6 \end{bmatrix}$ and $\begin{bmatrix} 7 & 9 & 5 \\ 8 & 3 & 5 \\ -8 & 1 & -6 \end{bmatrix} \begin{bmatrix} -5 \\ 0 \\ 8 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ -8 \end{bmatrix}$. What is an eigenvalue of A?

(d) [2 points] $A = \begin{bmatrix} 18 & -3 & 4 \\ -3 & 8 & 0 \\ 8 & 2 & 17 \end{bmatrix}$ and $\begin{bmatrix} 18 & -3 & 4 \\ -3 & 8 & 0 \\ 8 & 2 & 17 \end{bmatrix} \begin{bmatrix} -10 \\ 6 \\ 17 \end{bmatrix} = \begin{bmatrix} -130 \\ 78 \\ 221 \end{bmatrix}$. What is an eigenvalue of A?

(e) [2 points] $A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -\sqrt{3} \\ 2 \\ -\sqrt{3} \\ 1 \end{bmatrix} = \begin{bmatrix} -\sqrt{3} \\ 3 \\ -2\sqrt{3} \\ 3 \\ -\sqrt{3} \end{bmatrix}$. What is an eigenvalue of A?

5. Find eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$.

(a) [2 points] Solve the equation $\det(A - \lambda I) = 0$ for its three roots, λ_1, λ_2, and λ_3.

(b) [2 points] Solve the equation $(A - \lambda_1 I)\mathbf{x} = 0$ for an eigenvector associated with the eigenvalue λ_1.

(c) [2 points] Solve the equation $(A - \lambda_2 I)\mathbf{x} = 0$ for an eigenvector associated with the eigenvalue λ_2.

(d) [2 points] Solve the equation $(A - \lambda_3 I)\mathbf{x} = 0$ for an eigenvector associated with the eigenvalue λ_3.

(e) [2 points] For each eigenvector found in parts (b), (c) and (d), calculate the product $A\mathbf{x}$ and verify that $A\mathbf{x}$ is a scalar multiple of $\mathbf{x}$.

6. Find the eigenvalues of the following matrices and answer the questions below.

(a) [2 points] $A = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$

(b) [2 points] $A^T = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$

(c) [2 points] $A^T A = \begin{bmatrix} 5 & 3 \\ 3 & 9 \end{bmatrix}$

(d) [1 point] What is the sum of the eigenvalues of A?

(e) [1 point] What is the product of the eigenvalues of A?

(f) [1 point] What is the sum of the eigenvalues of $A^T A$?

(g) [1 point] What is the product of the eigenvalues of $A^T A$?