MATH 2418: Linear Algebra

Assignment 10 (sections 5.1, 5.2)

Due: <u>April 17, 2019</u>   Term: <u>Spring, 2019</u>

[First Name]   [Last Name]   [Net ID]

Suggested problems (do not turn in):Section 5.1: 1, 2, 3, 4, 5, 7, 8, 9, 12, 13, 15, 16, 17, 18, 21, 22, 23, 28; Section 5.2: 1, 2, 4, 5, 6, 7, 9, 12, 13, 16, 19, 20, 23, 24. Note that solutions to these suggested problems are available at *math.mit.edu/linearalgebra*

1. [10 points] Compute the determinants of the matrices A, B and AB.

$$A = \begin{bmatrix} 0 & 0 & -3 \\ -2 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 11 & 9 & 7 \\ 0 & 13 & 2 \\ 0 & 7 & 1 \end{bmatrix}$$

2. [10 points] Let 3×3 matrices A and B have determinants -1 and 8 correspondingly.

   (a) (3 points) Find determinant of $(4A^T)B^{-1}$.

   (b) (3 points) Find determinant of $B^2 A^{-1}$.

   (c) (3 points) Find a scalar $c \in \mathbb{R}$ such that $\det[(cA)B] = 1$.

   (d) (1 point) Find determinant of A^{-2017}.

3. [10 points] Compute the determinant of the matrix

$$B = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -2 & 0 & 0 & 3 \\ 3 & 0 & 4 & 0 \\ -4 & 2 & 0 & 0 \end{bmatrix}$$

4. [10 points] Let $A = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & 1/2 \\ 0 & -1/2 & \sqrt{3}/2 \end{bmatrix}$. Find the determinant of the matrix $A^T B$.

5. [10 points] Find the determinant of the matrices P, P^2 and P^3.

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

6. [10 points] True or False? Circle your answer and **provide a justification** for your choice.

   (a) **T F:** If matrix A has one of its diagonal elements equal to zero, then $\det(A) = 0$.

   (b) **T F:** Absolute value of the determinant of the orthogonal matrix equals to 1.

   (c) **T F:** If $\det B = \det B^{-1}$ then $B = I$.

   (d) **T F:** If $\det(2C) = \det(3C)$ then C is not invertible.

   (e) **T F:** There are no matrix D such that $\det(D^{-1}) = 0$.