MATH 2418: Linear Algebra

Assignment# 1

Due : Wednesday, 01/23/2019 Term :`Spring 2019`

[Last Name] [First Name] [Net ID] [Lab Section]

Recommended Text Book Problems (do not turn in): [Sec 1.1: # 1, 2, 3, 5, 6, 9, 10, 11, 12, 13, 14, 18, 19, 27, 28]

1. Let $\mathbf{u} = (2, 3, -1)$, $\mathbf{w} = (1, -1, 1)$, and $3\mathbf{u} - 2\mathbf{v} + 4\mathbf{w} = (-1, 2, 3)$. Find (a) $\mathbf{v}$ (b) $-2\mathbf{u} + 3\mathbf{v} - 5\mathbf{w}$.

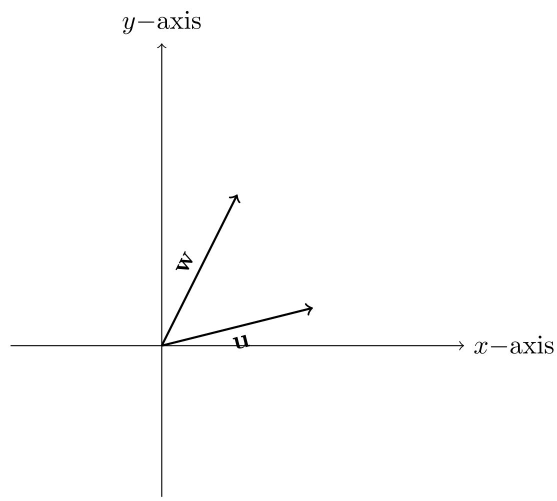
2. Given vectors **v** and **w** in diagram below, shade in all linear combinations $c\mathbf{u} + d\mathbf{w}$ for

   (a) $0 \leq c \leq 1$ and $-1 \leq d \leq 0$

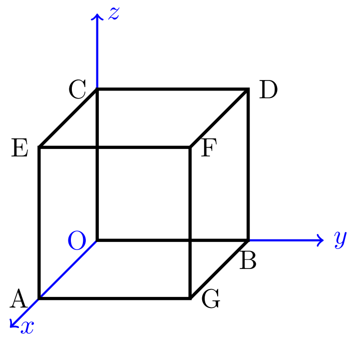
   (b) $0 \leq c \leq 1$ and $1 \leq d$

   (c) $0 \leq d \leq 1$ and $c \geq 1$.

   (You can use different shading styles in same picture for all three parts or can graph them separately in three different pictures)



3. Let $\mathbf{0} = (0,0,0)$, $\mathbf{i} = (1,0,0)$, $\mathbf{j} = (0,1,0)$, $\mathbf{k} = (0,0,1)$ be vectors in $\mathbb{R}^3$. Let P, Q, R, S, T, U be the center of faces $OAGB, OBDC, OAEC, GBDF, FECD, AGFE$ respectively of the unit cube in the figure below. Write down the following vectors as a linear combination of $\mathbf{i}, \mathbf{j}, \mathbf{k}$.



(a) $\overrightarrow{OP} =$

(b) $\overrightarrow{OQ} =$

(c) $\overrightarrow{OR} =$

(d) $\overrightarrow{OS} =$

(e) $\overrightarrow{OT} =$

(f) $\overrightarrow{OU} =$

4. Consider the unit cube as in the previous question. The face $OAGB$ can be described as a linear combination of $\mathbf{i}, \mathbf{j}, \mathbf{k}$ as

$$OAGB = \{c\mathbf{i} + d\mathbf{j} : 0 \leq c \leq 1, 0 \leq d \leq 1\}.$$

Geometrically describe the remaining five faces of the unit cube as a linear combination of $\mathbf{i}, \mathbf{j}, \mathbf{k}$.

(a) $OBDC =$

(b) $OAEC =$

(c) $GBDF =$

(d) $FECD =$

(e) $AGFE =$

5. Given three vectors $\mathbf{u} = (1, 1), \mathbf{v} = (1, -1)$, and $\mathbf{b} = (2, 4)$ in $\mathbb{R}^2$. Suppose $\mathbf{b}$ can be written as linear combination of $\mathbf{u}$ and $\mathbf{v}$ as

$$c\mathbf{u} + d\mathbf{v} = \mathbf{b}$$

(a) Write two equations in c and d corresponding to the vector equation $c\mathbf{u} + d\mathbf{v} = \mathbf{b}$.

(b) Solve the equations in part (a) for c and d.

(c) Express $\mathbf{b}$ as a linear combination of $\mathbf{u}$ and $\mathbf{v}$.

6. Let $\mathbf{u} = (1,1),\ \mathbf{v} = (1,-1)$, be 2 given vectors in $\mathbb{R}^2$. Let $\mathbf{b} = (p,q)$ be any vector in $\mathbb{R}^2$.

   (a) Write two equations in c and d corresponding to the vector equation $c\mathbf{u} + d\mathbf{v} = \mathbf{b}$.

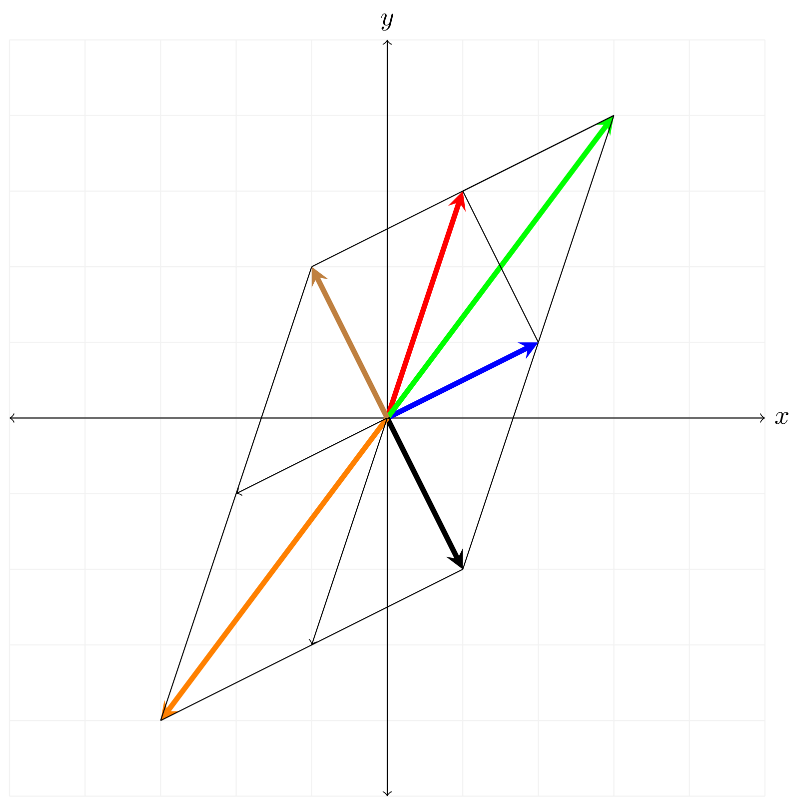
   (b) Solve the equations in part (a) for c and d.

   (c) Express $\mathbf{b}$ as a linear combination of $\mathbf{u}$ and $\mathbf{v}$ (if possible).

7. Let $\mathbf{u} + \mathbf{v} = (3, 4)$ and $\mathbf{u} - \mathbf{v} = (1, -2)$.

(a) Find $\mathbf{u}$ and $\mathbf{v}$

(b) Use the vectors $\mathbf{u},\ \mathbf{v}, -\mathbf{u}, -\mathbf{v},\ \mathbf{u} + \mathbf{v},\ (\mathbf{u} - \mathbf{v}),\ (-\mathbf{u} + \mathbf{v}),\ (-\mathbf{u} - \mathbf{v})$ to label the following figure.



8. Consider a clock of radius 2 unit.

   (a) Write down the 12 vectors in the component form that go from the center of the clock to the hours

$$1:00, 2:00, \cdots\cdots\cdots\cdots, 12:00.$$

   (b) What is the sum of those 12 vectors? Explain.

9. Consider the same clock as in previous question.

   (a) Write down the 12 vectors in component form that go from $3:00$ on the right to the hours

$$1:00, 2:00, \cdots\cdots\cdots, 12:00.$$

   (b) What is the sum of the 12 vectors? Explain

10. For each set of vectors in $\mathbb{R}^3$ given below. Describe geometrically the set of all linear combinations (a line or plane or all of $\mathbb{R}^3$).

    (a) $\{(0,1,-3),(0,-2,6),(4,2,-6)\}$

    (b) $\{(2,1,0),(1,1,1),(4,3,2)\}$

    (c) $\{(2,-3,1),(-4,6,-2),(-10,15,-5)\}$