MATH 2418: Linear Algebra

Extra Practice Problems (Sections 6.5 and 7.2)

Term: Spring 2019

- Section 6.5: 3, 4, 7, 10, 12, 18, 19
- Section 7.2: 2, 4, 7, 8

Solutions to the above problems are available at *math.mit.edu/linearalgebra*

More extra problems on positive definiteness and SVD of matrices:

1. Test positive definiteness for $S = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$.

Answer:

Its upper left determinants are 2, 3, and 4, all positive. So, the matrix S is positive definite.

2. Find all values of $b \in \mathbb{R}$ such that for $T = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & b \\ -1 & b & 2 \end{bmatrix}$ is positive definite.

Answer:

Its first two upper left determinants are 2 and 3, all positive. The determinant

$$\det(T) = -2(b-2)(b+1).$$

Thus, T is positive definite if and only if $-1 < b < 2$.

3. Find a full SVD of $A = \begin{bmatrix} -1 & 0 & -1 \\ 2 & 0 & 2 \end{bmatrix}$.

Answer:

$A = U\Sigma V^T$ with

$$U = \begin{bmatrix} -\frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix}, \qquad \Sigma = \begin{bmatrix} \sqrt{10} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad V = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}.$$

4. Find a full SVD of $A = \begin{bmatrix} -1 & -1 \\ 1 & -2 \\ -1 & -1 \end{bmatrix}$.

Answer:

$A = U\Sigma V^T$ with

$$U = \begin{bmatrix} -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}, \qquad \Sigma = \begin{bmatrix} \sqrt{6} & 0 \\ 0 & \sqrt{3} \\ 0 & 0 \end{bmatrix}, \qquad V = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$